Review Notes for Ch. 19 & beyond

19.1 • Flux concept: volume of water flowing through a net.

• Oriented Surface: normal vectors at each point on surface in a continuous way. Two possible directions to choose direction of positive flux.
For closed surfaces we default to outward orientation.

• Area Vector for a flat oriented surface
If $\hat{n}$ is a unit-normal in positive flux direction, ~~then and~~ then $\vec{A} = (\text{Area of surface})\hat{n}$

• If $\vec{F}$ is constant and S is a flat surface with area vector $\vec{A}$, then

$$\text{Flux of } \vec{F} \text{ through } S = \vec{F}\cdot\vec{A} \; \left(= \|\vec{F}\|\,\|\vec{A}\|\cos\theta\right)$$

• If $\vec{F}$ is not constant or S is not flat

$$\text{Flux of } \vec{F} \text{ through } S = \int_S \vec{F}\cdot d\vec{A} \quad \left(\text{note: } d\vec{A} = \hat{n}\, dA\right)$$

19.2 Flux Integrals for 3 Special Situations.

• if S is a portion of a surface $z = f(x,y)$, then $\boxed{d\vec{A} = -f_x\vec{i} - f_y\vec{j} + \vec{k}}$

• if S is part of a cylinder of radius R centered about the z-axis, then $\boxed{d\vec{A} = (\cos\theta\,\vec{i} + \sin\theta\,\vec{j})R\,dz\,d\theta}$

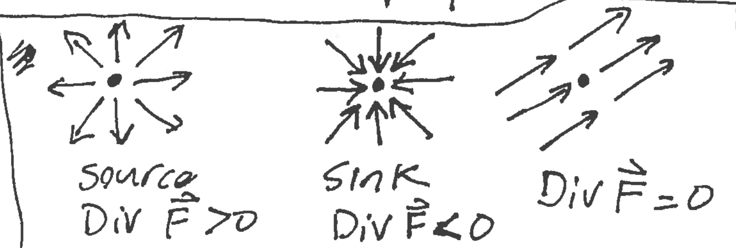
• If S is part of a sphere of radius R centered at (0,0,0), then

$$\boxed{d\vec{A} = \left(\sin\phi\cos\theta\,\vec{i} + \sin\phi\sin\theta\,\vec{j} + \cos\phi\,\vec{k}\right)R^2\sin\phi\, d\phi\, d\theta}$$

19.3 Divergence

- $\text{Div}\,\vec{F} = \text{Flux Density} = \lim_{\text{Volume}\to 0} \dfrac{\int_S \vec{F}\cdot d\vec{A}}{\text{Volume of } S}$

  $\text{Div}\,\vec{F} = \vec{\nabla}\cdot\vec{F}$



Since $\text{Div}\,\vec{F}$ = Flux density, we can approximate the flux ~~through~~ out of a small closed surface about (a,b,c) with Volume V by flux through $S \approx (\text{div}\,\vec{F}\,(a,b,c))(V)$

19.4 Divergence Theorem:

If W is a solid region whose boundary S is a piecewise smooth surface, and if $\vec{F}$ is a smooth vector field on an open region (no holes) containing W & S, then

$$\int_S \vec{F}\cdot d\vec{A} = \int_W \text{div}\,\vec{F}\,dV$$

"Flux of $\vec{F}$ ~~through~~ out of S = Integral of div $\vec{F}$ over Region enclosed by S."

20.1

3D curl $= \vec{\nabla}\times\vec{F}$ is the axis about which $\vec{F}$ would cause a magnetically suspended ping pong ball to rotate.

The direction of $\vec{\nabla}\times\vec{F}$ is given by the right hand rule, and $\|\vec{\nabla}\times\vec{F}\|$ relates to how quickly the ball would rotate

(More formally, Curl $\vec{F}$ = (Circulation density of $\vec{F}$ about $\hat{n}$) $\hat{n}$ where $\hat{n}$ is the axis the ball would spin about.

The circulation density of $\vec{F}$ about a point (a,b,c) = $\text{Circ}_{\hat{n}}\,\vec{F}(a,b,c) = [\text{Curl}\,\vec{F}_{(a,b,c)}]\cdot[\hat{n}]$ about a path C with "orientation vector" $\hat{n}$

20.2

Stokes' Theorem

If S is a smooth, oriented surface with a piecewise smooth, oriented boundary C, and if $\vec{F}$ is a smooth vector field on an open region containing S & C, then

$$\int_C \vec{F} \cdot d\vec{r} = \int_S \operatorname{curl} \vec{F} \cdot d\vec{A}$$

circulation of $\vec{F}$ about C = Flux of curl through S